Bideford College

Part of the Athena Learning Trust

Transition to A level Mathematics resources: Essential Skills

Many students say that they find the initial transition from GCSE to A level challenging. These resources focus on key skills that will be used across the whole spectrum of AS and A level Mathematics. Each section includes:

- Skills checks
- A chance to practise and explore
- Some extra ideas that you may want to investigate further

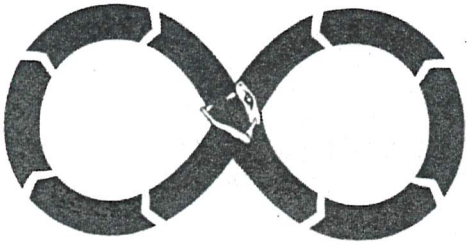
There are eight sets of resources, and each set should provide about 2 hours of work. If you really get engaged by the enrichment activities, you may want to spend longer than this.

We hope you find these resources useful, that they add depth to your understanding, and that they help you make a smooth and successful transition to AS and A level Mathematics.

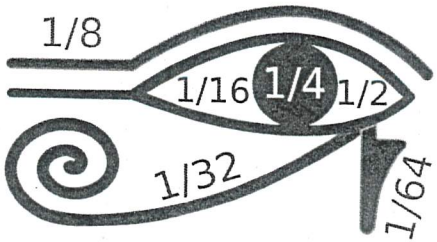
Contents

Section 1 - Fractions

Did you know ?

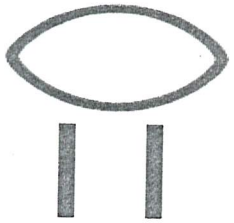


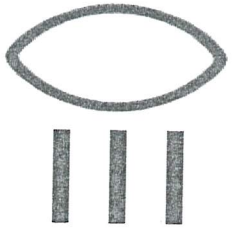
Ouroboros

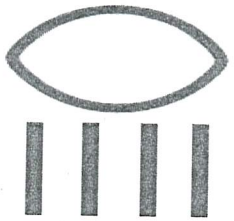


The eye of Horus

5000 years ago the Egyptians used fractions but could only write unit fractions, fractions where the numerator is 1.

$\frac{1}{2}$ = 

$\frac{1}{3}$ = 

$\frac{1}{4}$ = 

All other fractions were written as sums of unit fractions but more about that later……

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Fractions 2

1. Calculate $2\frac{1}{7} + 1\frac{1}{5}$
   Give your answer as a mixed number in simplest form

2. Simplify $\frac{4a}{5} \times \frac{7b}{3}$

3. Work out $\frac{19}{24} - \frac{3}{8}$
   giving your answer in simplest form

4. Find the mean of $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ and $\frac{1}{6}$
   give your answer in simplest form

5. A full glass of water can hold $\frac{1}{6}$ of a bottle of water.
   How many glasses can be filled by $2\frac{1}{5}$ bottles?

6. A water tank is $\frac{2}{3}$ full
   40 litres of water are taken from the tank
   The tank is now $\frac{1}{2}$ full
   What fraction of the tank was removed ?

7. Which of these has the largest value

   $\frac{1}{2} + \frac{1}{4}$ $\frac{1}{2} - \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{4}$ $\frac{1}{2} \div \frac{1}{4}$ $\frac{1}{4} \div \frac{1}{2}$

8. Simplify $\frac{a}{b} + \frac{b}{c}$

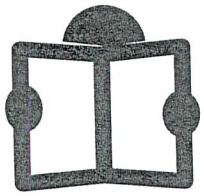
You can do this for fun - or move on if you correctly completed Skills check 1.

Fractions of a 1000

What is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of 1000 ?

If you like this challenge then you can play an interactive challenge on Nrich - Level 4!

Still want more?



Read some more about interesting fractions and how fractions are everywhere!

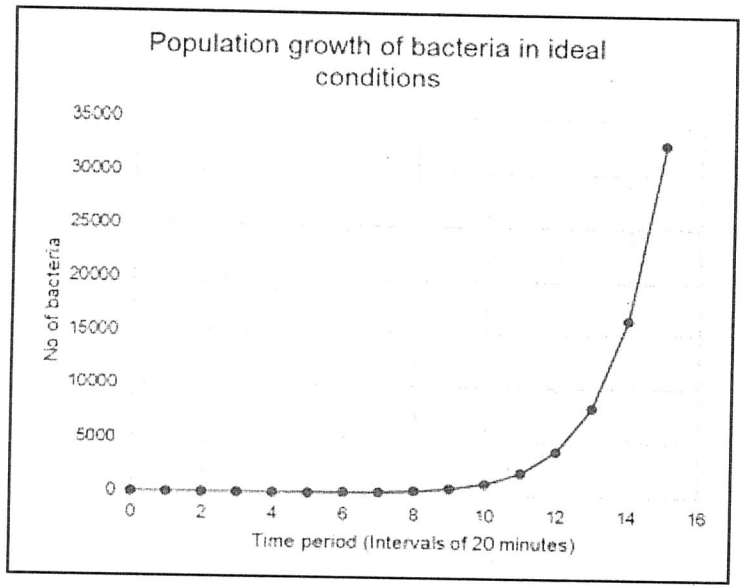
Discover more about fractions and series. This task explores what happens when we add fractions repeatedly.

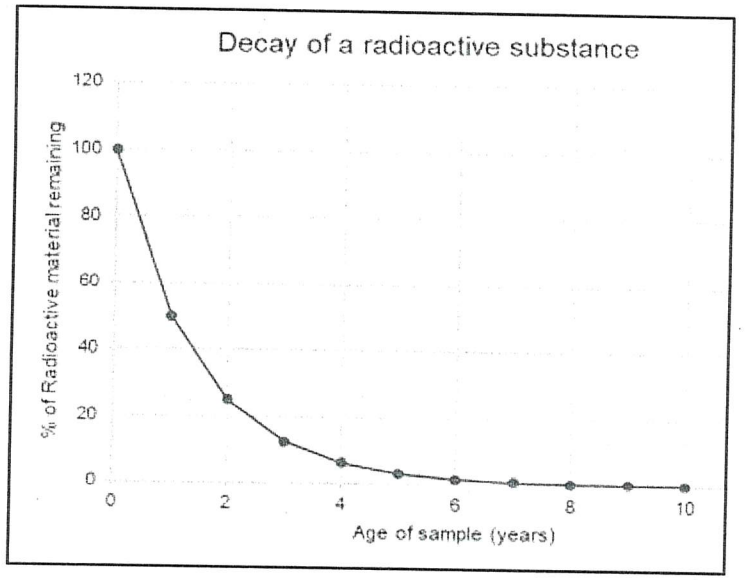
Watch this video to find out one way that fractions connect biology and mathematics.

Section 2 - Indices

Did you know?

Indices are also referred to as Exponents





e.g. $2^3 = 8$ ← 3 is the 'exponent'

$2^3 = 2 \times 2 \times 2$

It tells us how many times a number is multiplied by itself

This is where exponential graphs come from!

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Indices 2

Simplify the following:

1. $t^5 \times t^4 =$

2. $\frac{8^7}{8^2} =$

3. $(3^4)^2 =$

4. $\frac{5^7 \times 5}{(5^3)^3} =$

5. $8^{\frac{1}{3}} =$

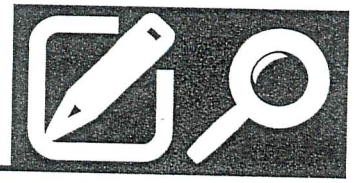
6. $y^0 =$

7. What is 3^{-4} ?

8. What is $\left(\frac{2}{3}\right)^{-2}$

You can do this for fun - or move on if you correctly completed Skills check 1.

Roots and Indices Maze



Can you find the route to the opposite side of the table?

- Begin in the highlighted box
- Move horizontally or vertically one box at a time… no diagonal moves allowed!
- You may only land on boxes which are equivalent in value to the highlighted one

$2^6 \times 2^3$	$3^2 \times 2^3$	$(\sqrt{16})^2$	$(2^3)^3$	$8^3 \div 8$	$4^4 \times 4^{-3}$	$(\sqrt[3]{8})^4$	8×4^2
$\sqrt{8^3}$	$(2^3)^2$	$8^7 \times 8^{-5}$	4^3	$2^{-2} \times 2^7$	64^0	$2^5 \times 2^3$	$4^7 \div 2^3$
$(\sqrt{64})^3$	8^2	$2^2 \times 2^3$	$2^3 \times 2^3$	$(2^3)^3$	$(\sqrt[3]{8})^6$	$4^6 \times 4^{-3}$	$2^2 \times 4^2$
2^6	$(\sqrt{64})^2$	$4^6 \times 4^{-2}$	$(\sqrt{16})^3$	$(2^2)^4$	$8^3 \div 2^3$	$2^{-3} \times 2^7$	$(2^2)^4$
3^5	$2^6 \times 2^1$	8^3	$4^5 \div 2^4$	$(-4)^{-3}$	$(2^2)^3$	$(\sqrt{8})^3$	$4^6 \div 2^6$
$4^3 \times 4^{-3}$	$(2^5)^1$	$(\sqrt[3]{64})^2$	$2^3 \times 8$	$2^{-1} \times 2^7$	$(\frac{1}{4})^{-3}$	16^2	64

Hint: What is the value of 2^6

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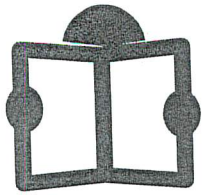
Still want more?



Read how maths is used in different careers. For indices and exponential growth check out *Population Dynamics*, *Epidemics Analysis* and *Carbon Dating* in particular.

Discover the power of indices! Here you will see how they could be used to knock down very tall buildings!!

Watch this Numberphile video and learn how to impress friends and family by finding the fifth root of a number in the blink of an eye.

Section 3 - Surds

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Did you know ?

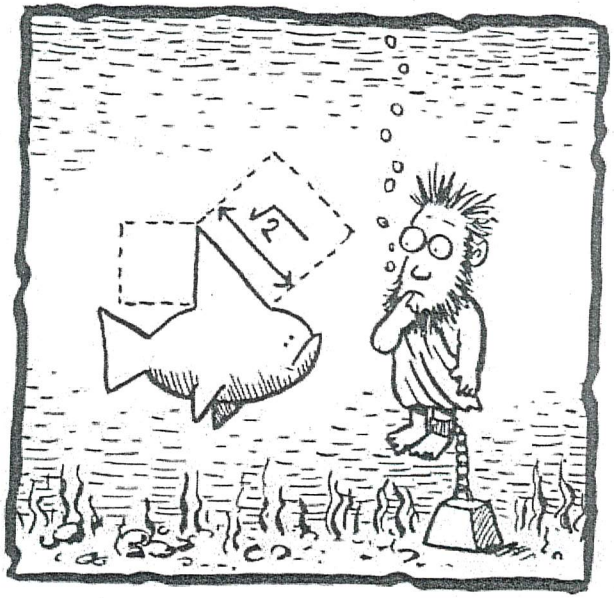
Maths can be murderous!

You will have heard of Pythagoras and his theorem but have you heard of Hippasus who was one of his followers?

Pythagoreans preached that all numbers could be expressed as the ratio of integers – i.e. fractions.

Hippasus is sometimes credited with the discovery of the existence of irrational numbers – proving it for $\sqrt{2}$.

Following which, he was drowned at sea!



https://www.flickr.com/photos/28698046@N08/21275364908/

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Surds 2

1. Simplify $\sqrt{d} + 6\sqrt{d} - 3\sqrt{d}$

2. Simplify $2\sqrt{b} \times 4\sqrt{3}$

3. Simplify fully $(4\sqrt{5})^2$

4. Write $\sqrt{75} + \sqrt{48} - 2\sqrt{12}$ in the form $k\sqrt{3}$

5. Simplify $\frac{\sqrt{125} - 2\sqrt{20}}{\sqrt{5}}$

6. Rationalise the denominator of $\frac{2\sqrt{2}}{\sqrt{5}}$

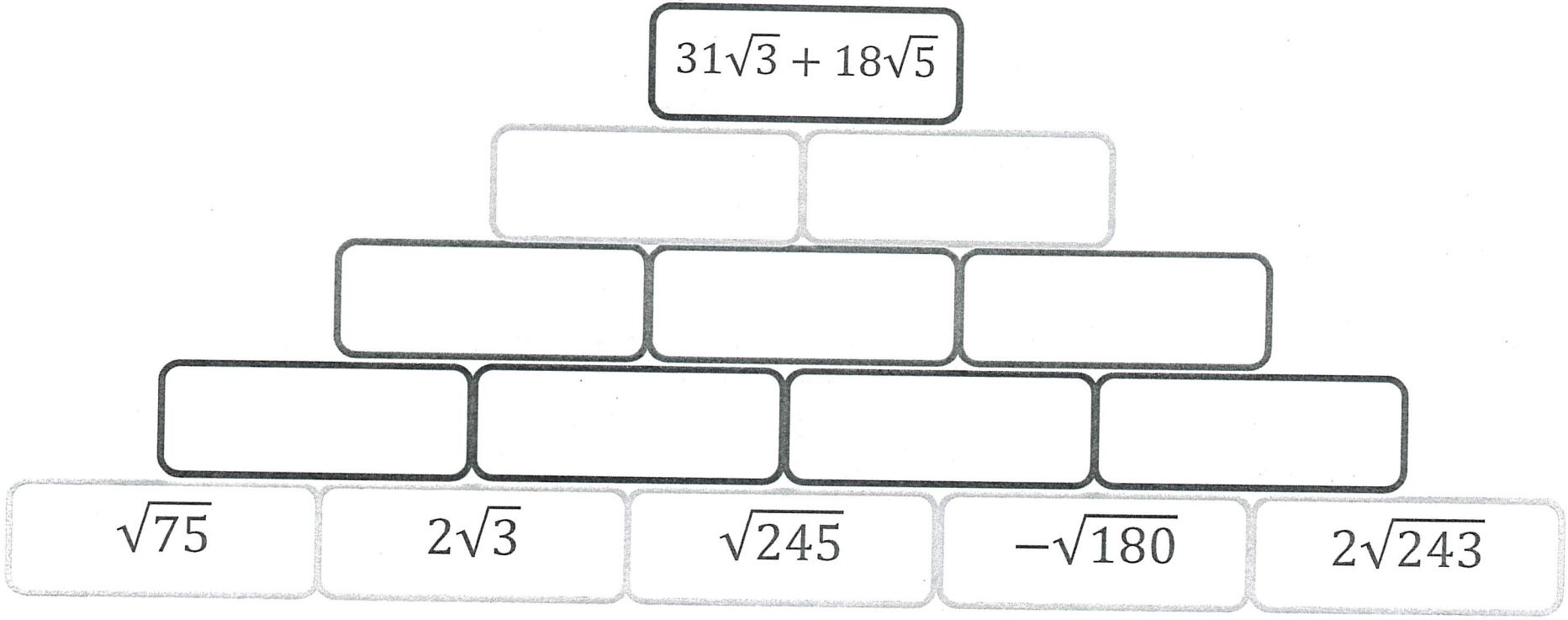
7. Evaluate $\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{6}}$

   Give you answer in simplest form.
   Rationalise the denominator.

8. A triangle has a base of $3\sqrt{2}$ and a perpendicular height of $5\sqrt{8}$.
   Calculate the area of the triangle.

Another brick in the wall

Complete the empty boxes in the pyramid.
Each box is the sum of the two boxes directly below it.



*Hint: You may need to simplify some of the surds in the bottom row to get started.

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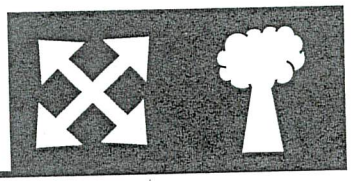
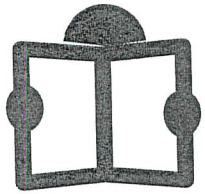
Still want more?





Read about how Irrational numbers can "Inspir-al" you! It's where mathematics and art meet!

Discover the proof, that $\sqrt{2}$ is irrational – without getting murdered like Hippasus.

Watch this video to find out more about the special properties of A4 paper and discover what makes $\sqrt{2}$ one of the most popular surds of all time.

Section 4 – Expanding brackets

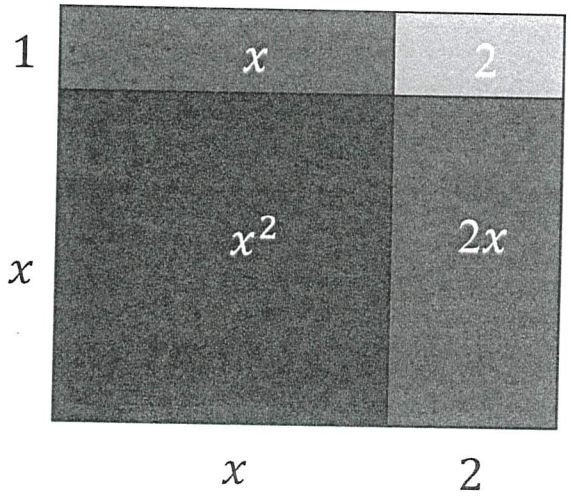
Have a think…

… about $(x + 2)(x + 1)$

Formal Method

$$(x + 2)\,(x + 1)$$
$$= x\,(x + 1) + 2(x + 1)$$
$$= x^2 + x + 2x + 2$$
$$= x^2 + 3x + 2$$

Geometrical Representation



$$= x^2 + 3x + 2$$

Grid Method

	x	$+2$
$+1$	x	2
x	x^2	$2x$

$$= x^2 + 3x + 2$$

- This expression expands to give 4 terms…which simplify to 3 terms.
- How many terms are in the unsimplified expansion of $(x + 3)(x + 4)(x + 5)$?
- Be prepared to explain your thinking…

The story so far.....

1. Expand and simplify
$$(2x + 3)(3x - 5)$$

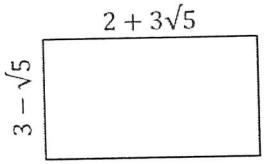
2. Write $(x + 3)^2 - 4$ in the form $ax^2 + bx + c$

3. Expand and simplify
$$(2a + 2)(3x - 4a + 3)$$

4. Expand and simplify
$$3x(x - 3)(x + 5)$$

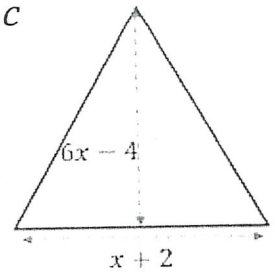
5. Evaluate (no calc allowed)
$$\left(2 + \frac{1}{3}\right)\left(2 - \frac{1}{3}\right)$$

6. Find the area of this rectangle

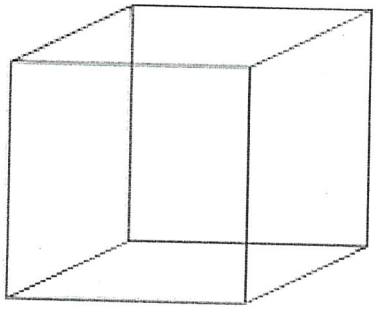


7. Expand and simplify
$$(5 - 4x)(3x + 6) + (5x - 2)(3 + 4x)$$

8. Find the area of the triangle and write it in the form $ax^2 + bx + c$



Getting bigger



Here is a cube with side lengths of x cm

The cube is going to have its lengths increased in one of three ways

Method A	Method B	Method C
Each side is increased by 2 units	One side is increased by 3 units, one side is increased by 2 units, and one side is increased by 1 unit	One side is increased by 5 units, one side is increased by 2 units, and one side is decreased by 1 unit

Can you prove which of the new solids will have the largest volume?

Summary and Review

1. Expand and simplify
$$\left(\frac{1}{3}x+\frac{1}{9}\right)\left(3x-\frac{2}{3}\right)$$

2. Expand and simplify
$$(x+1)(x+2)(x+3)$$

3. Expand and simplify
$$(x-3)(x+2)^2$$

4. Expand and simplify
$$(2-\sqrt{3})(1+\sqrt{3})(1-\sqrt{3})$$

5. Find the volume of a cube with side length
$x-4$

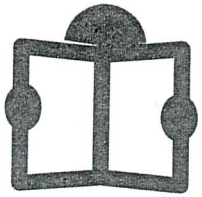
6. Expand and simplify
$$(x^2-2)(x^2+2)(x+1)$$

7. Write $\left(\sqrt{y}+\sqrt{8y}\right)^2$ in the form $a+b\sqrt{2}$.

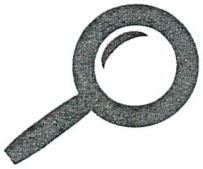
Given that $\left(\sqrt{y}+\sqrt{8y}\right)^2 = 54+b\sqrt{2}$.
Find values for y and b.

8. Simplify $\frac{(x-1)(x+2)}{(x+3)} - \frac{4}{2x+1}$

Still want more?



Read more about Pascal's triangle, interact with it and find out more about it's heritage and who really discovered it first!



Discover more expansions linking to geometrical representations. You'll find a hint and a potential solution from other students to help you too.

Watch this video and encounter the almost endless amount of number patterns contained within Pascal's triangle.

Section 5 – Factorising

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Did you know?

?

Substitute $x = 9$ into the following two expressions

$$x^2 + 3x + 2$$

and

$$(x + 2)(x + 1)$$

What do you notice?

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Further Factorising 2

Factorise the following fully:

1. $x^2 + 6x - 7$
2. $y^2 + y - 12$
3. $y^2 - 11y + 28$
4. $t^2 - 7t - 18$
5. $k^2 + 9k + 20$
6. $x^2 + x - 56$
7. $p^2 - 25p$
8. $x^2(3x - 4) + (4 - 3x)$

You can do this for fun - or move on if you correctly completed Skills check 1.

Trickier Quadratics

- Try factorising these expressions
- You might want to try the grid method.

1. $3x^2 - 10x - 8$

2. $2x^2 - 7x + 6$

3. $4y^2 + 20y + 9$

4. $6x^2 - 13x - 8$

5. $20x^2 + x - 12$

*Hint. There are some partially filled grids on the next slide if you want to use them

Without a calculator

What is the value of each of the following?

calculators not allowed

$$9^2 - 1^2$$

$$99^2 - 1^2$$

$$999^2 - 1^2$$

Hints available on the next slide

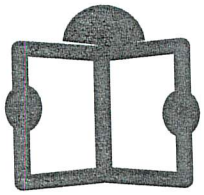
Still without a calculator

Without using a calculator, find the value of

$$\frac{122 \times (122^2 + 4 \times 123)}{124} - \frac{124 \times (124^2 - 4 \times 123)}{122}$$

Hints available on the next slide

Still want more?



Explore the history of mathematics with this interactive historical timeline -in particular look for at Al-Khwarizmi. Can you find a famous artist and a mathematician whose triangle you met in the Expanding topic?

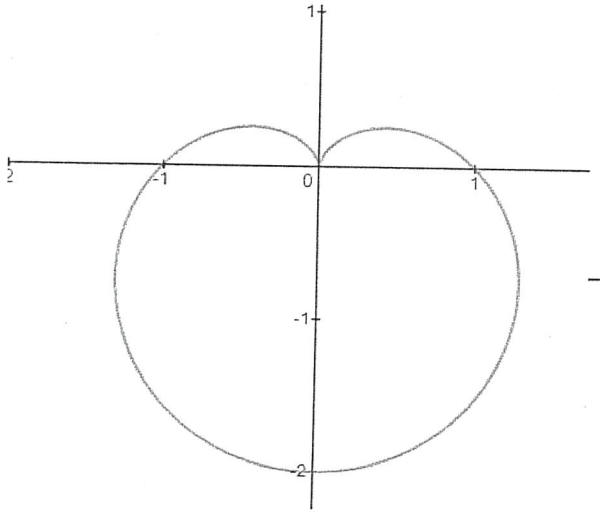
Discover how you can use factorising quadratics and apply it to higher powers by this neat trick shown in this nrich task.

Watch how you can apply difference of two squares to a fun numerical problem.

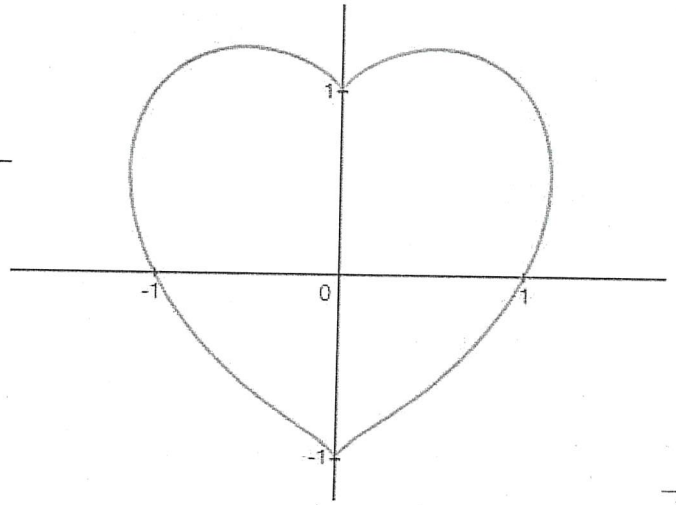
Section 6 – Rearranging

Did you know?

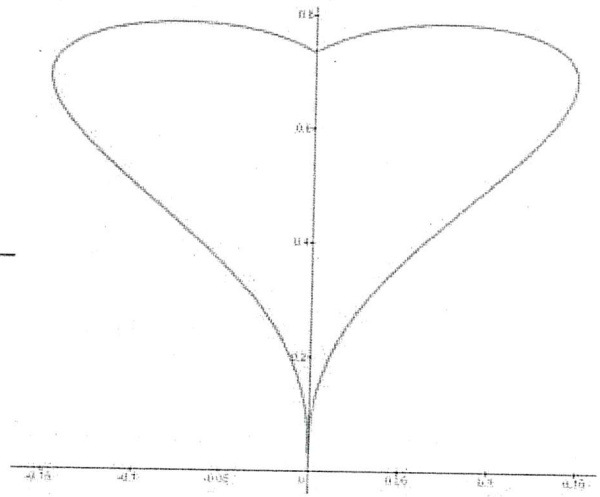
- Being able to express equations in different forms gives us different information
- Later we'll be looking at information needed to sketch graphs
- If you continue your maths studies to A Level Further Maths, you will draw graphs such as these



$r = 1 - sin\theta$



$(x^2 + y^2 - 1)^3 - x^2y^3 = 0$



$x = \sin(t)\cos(t)\log(|t|)$
$y = |t|^{0.3}\sqrt{\cos(t)}$

Further Factorising 2

1. Make y the subject of

$$xy + 6 = 7 - ky$$

2. Find an expression for the area of a rectangle with length, $(y - x)$ and width, $(x - 2)$

3. Rewrite your expression in Q2 to have y expressed in terms of A and x

4. Make y the subject of $\frac{4}{y} + 1 = 2x$

5. Displacement can be expressed as

$$s = ut + \frac{1}{2}at^2$$

Express a in terms of s, u and t

6. Make y the subject of $\sqrt{by^2 - x} = D$

7. The area of a trapezium has formula

$$A = \frac{1}{2}\left(\frac{a+b}{h}\right)$$

Express h in terms of A, a and b

8. Make t the subject $b(t + a) = x(t + b)$

Mean squares

- Take two positive values greater than 1
- Find the mean of the two values
- Square it

Then

- Take the same two values
- Square them
- Find the mean of the squares

Which value is greater?

Is this always true?

Can you prove it?

- Try out several examples
- Is one expression always bigger than the other?
- Next try using x and y instead.
- If you subtract one expression from the other, can you work out if it's positive or negative?

Still want more?

Discover how using a graphing app such as Desmos or GeoGebra can help you gain insight into circles, tangents and graphs in general. Gain skills useful for A level maths.

Watch a TED talk from Dr Hannah Fry which trys to answer the question "Is life too complex?" You will see that you can actually write equations that model human behaviour!

Section 7 – Solving quadratics

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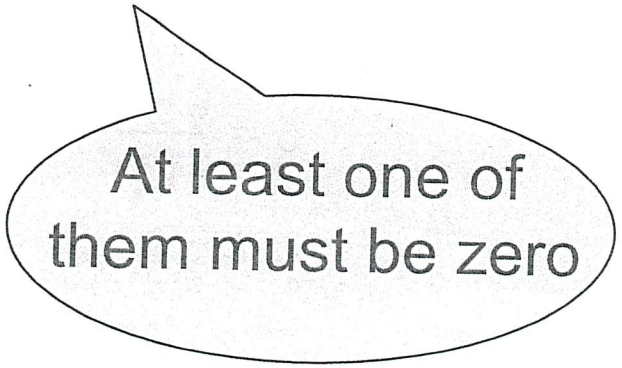
Did you know?

I have picked two numbers that multiply to make zero.

What can you say about my numbers?



This is useful when using factorising to solve equations.

If $a \times b = 0$, then either $a = 0$ or $b = 0$ (or both!)

Historically zero wasn't accepted as a number until relatively recently!

Solving with Quadratics 2

Solve the following

1. $x^2 - 4x - 12 = 0$

2. $x^2 - x = 6$

3. $2x^2 - 11x + 12 = 0$

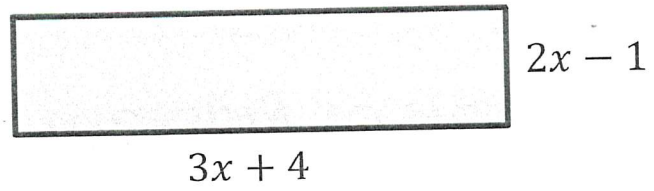
4. $6x^2 + x - 12 = 0$

5. $3 + 2x - x^2 = 0$

6. $x^2 - 4x - 1 = 0$

7. $\frac{8}{x+2} - \frac{14}{x-3} = 9$

8. The area of this rectangle is $30m^2$



a) Show that $6x^2 + 5x - 34 = 0$

b) Find any possible values for x

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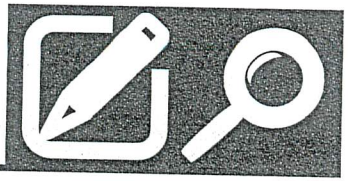
Simultaneously



A rectangle has length $(a + b)$ and width $3a$.

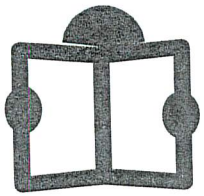
The area is $60cm^2$ and perimeter is $32\ cm$.

Calculate, algebraically, the possible values for a and b.

In how many places does the line $y = 2x + 2$ intersect the circle $(x + 2)^2 + y^2 = 25$?

What are the co-ordinates of these intersections?

Still want more?



Read about the history of Quadratic equations and how there are 101 uses for them!

Discover what is meant by a conic section and what on earth quadratics have to do with them.

Watch this video if you have ever been told that there are no solutions to a particular quadratic equation – because there are! They are not real though - welcome to imaginary maths! You can try a question for yourself here.

Section 8 – Graph sketching

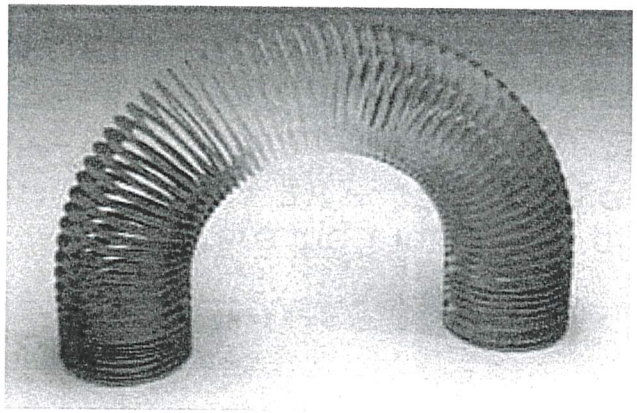
amsp®

Did you know?

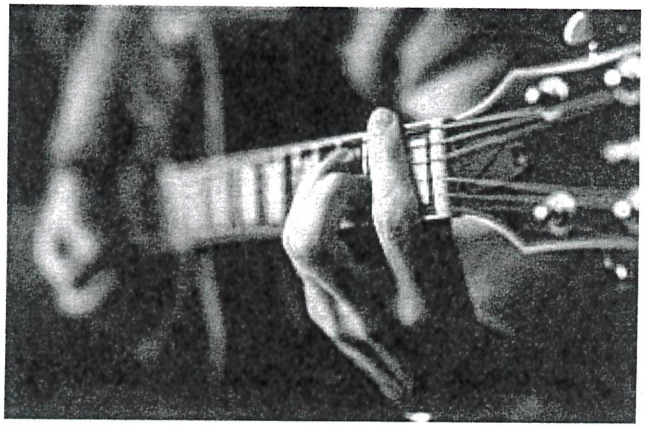
?

Trigonometric functions can be used to model many things that repeat over a time period, for example

Tides



Springs



Harmonic Strings

Daylight

amsp Sketching Other Graphs 1

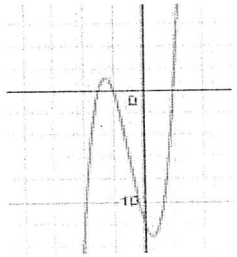
1. What is the mathematical name for the graph of $y = \frac{1}{x}$?

2. What are the maximum and minimum values for the graph $y = cos\theta$?

3. Sketch the graph of $y = 2^x$.
   Label the y and x intercepts

4. Using a sketch of the graphs
   $y = \frac{1}{x}$ and $y = x$
   show how many solutions there will be to the equation $\frac{1}{x} = x$

5. What is the name for this type of graph?

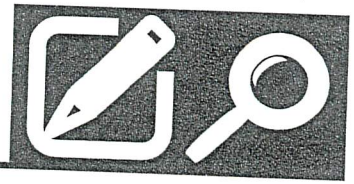


6. What is the y intercept of the graph
   $y = (x + 2)(x - 3)(x + 5)$?

7. What are the x intercepts of the graph
   $y = (x + 2)(x - 3)(x + 5)$?

8. Sketch the graph of
   $y = (x - 3)(x + 2)(x + 5)$

amsp Sketching more than graphs



Find the shortest distance between the following curves:

$$x^2 + y^2 = 9$$
$$y = x^2 + 7$$

A car is initially travelling at 300m/min, it speeds up over a 20 second interval with a constant acceleration to achieve a speed of 400m/min.
It travels at this speed for 3 minutes before slowing to a stop via constant de-acceleration over a period of 30 seconds.

a) What is the car's average speed for the first 20 seconds of travel?
b) What is the car's deceleration?

A square is placed inside a circle (C_1) so that the corners perfectly touch the circle's circumference.
Another circle (C_2) is now placed inside this square so that its circumference perfectly touches the square's sides.

What is the ratio of the lengths of the radius of C_1 and the radius of C_2 ?

Hint: Assume C_2 has a radius of 1 unit

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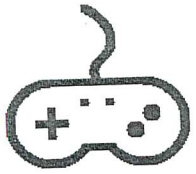
Still want more?

Read about Euclid's Axioms and discover how they might be used in this interactivity. Sketches and diagrams help with more than just questions about graphs!



Play 'Euclidea' to explore more about Euclidean Geometry and constructions.

Watch this video to see how you can 'graph' art! To see all the finalists in the Desmos Art competition (and get inspiration to enter it yourself in the future) click here.

Website links for further reading and investigation

Fractions

https://undergroundmathematics.org/thinking-about-numbers/fractions-everywhere/download/fractions-everywhere.pdf

https://meiassets.blob.core.windows.net/amsp-uploads/uploads/files/NA9__2_.pdf

https://www.youtube.com/watch?v=hLGDJFGAmic

Indices

https://mathigon.org/applications

https://mathigon.org/course/exponentials/carbon-dating

https://www.youtube.com/watch?v=5JCm5FY-dEY&t=4s

https://www.youtube.com/watch?v=y8acoaakvPM

Surds

http://www.ldlewis.com/Teaching-Mathematics-with-Art/documents/MTMS2007-04-442a.pdf

https://plus.maths.org/content/maths-minute-square-root-2-irrational

https://www.youtube.com/watch?v=5sKah3pJnHI

Expanding brackets

https://mathigon.org/course/sequences/pascals-triangle

https://nrich.maths.org/762/

https://www.youtube.com/watch?v=0iMtlus-afo

Factorising

https://mathigon.org/timeline

https://nrich.maths.org/2286

https://www.youtube.com/watch?v=LkIK8f4yvOU#action=share

Re-arranging

https://www.desmos.com/

https://www.ted.com/talks/hannah_fry_is_life_really_that_complex#t-267656

Solving quadratics

https://plus.maths.org/content/101-uses-quadratic-equation

https://mathigon.org/course/circles/conic-sections

https://www.bbc.co.uk/programmes/p058y4cy

Graph sketching

https://mathigon.org/course/euclidean-geometry/axioms#euclid

https://www.euclidea.xyz/

https://www.youtube.com/watch?v=MWmnXUXfFOg